Critical Link Detection and Ranking in Transportation Networks
via Day-to-Day Road Capacity Uncertainty

Jian Li, M.Sc. (Corresponding Author)
Graduate Research Assistant
Rutgers Intelligent Transportation Systems (RITS) Laboratory
Department of Civil and Environmental Engineering
Rutgers, the State University of New Jersey
623 Bowser Road,
Piscataway, NJ 08854-8014 USA
Tel: (732) 445-0576 x119
Fax: (732) 445-0577
E-mail: jianli@rci.rutgers.edu

Kaan Ozbay, Ph.D
Professor and Director
Rutgers Intelligent Transportation Systems (RITS) Laboratory
Department of Civil and Environmental Engineering
Rutgers, the State University of New Jersey
623 Bowser Road,
Piscataway, NJ 08854-8014 USA
Tel: (732) 445-2792
Fax: (732) 445-0577
E-mail: kaan@rci.rutgers.edu

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ABSTRACT

Critical link analysis in transportation planning is an important issue for public officials in order to avoid unnecessary economic costs. However, the analysis presents numerous challenges for capturing the impacts of uncertain events. This study proposes an analytical framework and an efficient solution procedure for link criticality evaluation, which considers the impact of uncertain events. Link capacity is considered as a multi-status variable based on day-to-day roadway traffic conditions and a sampling technique is used to generate realizations of network capacity values. With different capacity realizations, traffic demand is repeatedly assigned on the network, and the assignment results are analyzed by using several statistical indices. A case study based on a portion of the New Jersey roadway network is presented to verify the procedure.
INTRODUCTION
Recently, road network vulnerability analysis has been highlighted due to the increasing risks of uncertain events, such as natural disasters, incidents/accidents, etc. Uncertain events can result in short or long-term disruptions, particularly causing loss or partial blockage of certain heavily traveled network links. It was observed during the Hurricane Rita evacuation (1) that “an estimated three million people evacuated the Texas coast, creating colossal 100-mile long traffic jams that left many stranded and out of fuel; inefficient use of road capacity and the effects of ill-planned evacuation, resulting in disorganized movement of people.”

Identification of the critical links in a transportation network is an important part of road network vulnerability analysis. This problem is concerned with finding the links that result in severe deteriorations of network performance (e.g. total users’ travel time), when degradable. In the case of a specific application for transportation facilities assessment, most studies use an enumeration method (complete sampling): set scenarios that each assumes a certain link is degradable, repeatedly assign traffic demand, and compare network performance. The links which cause severe network performance deteriorations are the critical links. When considering link degradation, current studies usually assume binary status: fail or not. This assumption is reasonable for emergency situations, such as earthquakes or hurricanes, however, not for daily traffic conditions. Daily uncertain events may simultaneously result in different links degrading at different levels. In other words, the link operational status is multiplex, and degradation happens simultaneously for different links under day-to-day traffic conditions. Thus, it is necessary to adopt a more practical approach to capture the link criticality in degradable transportation networks under the concurrence of various daily causes.

The main objective of this study is to detect and rank critical links in transportation network under the risks of daily uncertainty events. We attempt to incorporate the analytical formulation in a regional transportation planning model with a realistic day-to-day, risk-related dataset. The risks of uncertain events are incorporated through the combination of frequency distributions of roadway capacity reductions. In order to represent day-to-day roadway conditions, we extend the traditional static traffic assignment model by considering capacity as a random variable. A sampling-based method is used for uncertainty and sensitivity analysis to determine the critical links. Total users’ travel time is used as the performance measure and the rank transformation technique is employed to linearize the nonlinear relationship between capacity and link travel time. Results are compared in terms of following statistical indices: rank correlation coefficients (RCCs), standardized rank regression coefficients (SRRCs) and partial rank correlation coefficients (PRCCs).

The paper is organized as follows: Section 2 reviews the literature related with critical-link determination in a degradable transportation network. Section 3 presents the methodology, including mathematical formulation, sampling-based method, and sensitivity analysis measures. In Section 4, a case study with a portion of the New Jersey roadway network is presented. The final section provides conclusions and opportunities for future research.

LITERATURE REVIEW
The problem addressed in this study is related with previous theoretical work on the most vital arc or edge problem. In detail, the problem is finding the arc or edge that on its removal results in maximum deterioration of network performance. The solution methods include graph theory and game theory. In graph theory, a general performance measure can be the increase of shortest path length (2-4). For example, (2) and (3) formulated the most vital arcs problem as determination of
the subset of arcs whose removal from the network resulted in the greatest increase in the shortest path length. The measure of the performance of a network in (4) was the increase of the distance between the origin nodes and sink nodes in a maximum flow graph. In game theory approach, (5) considered the situation where a network “spoiler” seeks to disrupt the network to maximize user costs (by choosing the link that causes the maximum impact), while users try to minimize their costs (by adjusting their routes according to the expected link costs). The results of the game were therefore worst-case link failure probabilities, which could be used to find the upper-bound impact of link degradation. Applications of this approach were described in (6) and (7).

For the specific application in the assessment of transportation facilities, most studies use bi-level formulations to solve the problem. At the lower level, assign vehicles to achieve the goal of user equilibrium or system optimality; at the upper level, critical links that maximize network performance deteriorations as a result of their removal are identified. The procedure mainly depends on two issues: the assignment methodology and performance measure.

The assignment methodology can be generally categorized as either static or dynamic traffic assignment. Static assignment assumes that traffic is in a steady status, and the time to traverse a link depends only on the number of vehicles on that link. Because of its simple mathematical formulation and solution procedure, static assignment is widely applied for link criticality evaluation on a regional network scale. Dynamic assignment can successfully represent the time-varying nature of the congestion during different times of the day, and help to understand travelers’ responses to time-varying transportation system operations. However, compared with static assignment, dynamic assignment requires more data-support and computation cost. Thus, dynamic assignment is more suitable for evaluating simple networks or arterial analysis considering users’ behavior (see (8) as an example).

Performance measure can be separated into two categories: accessibility and economic measures. Accessibility refers to the ‘ease’ of reaching opportunities for activities and services, and can be used to assess the performance of an urban transportation system (9-10). Economic measures refer to the cost of disruption due to the degradable critical links, which usually focus on travel time (11-15). In detail, alternative definitions and measures are summarized in (16).

**RESEARCH METHODOLOGY**

**Mathematical Formulation**

Static traffic assignment has found significant application since (17). Typically, two types of traffic assignments are performed: User Equilibrium (UE) - which assumes that users equilibrate such that each takes the shortest path, and System Optimum (SO) - which estimates link flows based on some system-wide objective (e.g., minimization of travel time). From (18) the formulation for each is:
SO : Min \( \sum_a x_a c_a (x_a) \) or UE : Min \( \int c_a (\omega) d\omega \)

\[
\begin{align*}
\text{s.t.} \quad & \sum x_k^rs = q_{rs} \quad \forall r,s \\
& h_k^rs \geq 0 \quad \forall r,s \\
& x_a = \sum_r \sum_s \sum_a h_k^rs \delta_{a,k} \quad \forall a \\
\end{align*}
\]

(1)

Where \( q_{rs} \) is the OD flow from origin \( r \) to the destination \( s \), \( h_k^rs \) is the flow on the path \( k \) from \( r \) to \( s \), \( \delta_{a,k} \) is a binary value indicating that link \( a \) exists on path \( k \) between \( r \) and \( s \), \( x_a \) is the flow on link \( a \), and \( c_a \) is the cost of link \( a \).

Traditionally, the adequacy of a road network is evaluated based on deterministic network capacity and origin-destination demand. In order to model the uncertainty events impact on link capacity, sensor data is more useful (19). When data is unavailable, the link status may be represented by the probability function as below:

\[
C_{ir} = f(S_{i1}, S_{i2}, \ldots, S_{im})
\]

(2)

Where \( C_{ir} \) is the realistic capacity of link \( i \); \( S_{i1}, S_{i2}, \ldots, S_{im} \) are uncertain events. For example, considering accidents and extreme weather conditions, the link capacity can be simply represented as:

\[
C_{ir} = (1-\alpha_{ia})(1-\alpha_{iw})C_i
\]

(3)

Where \( C_i \) is the recommended link capacity value in HCM 2000 (20). \( \alpha_{ia} \) and \( \alpha_{iw} \) are capacity reduction coefficients of accident and weather status, respectively. The link capacity distribution can be obtained by repeatedly calculating (3) for different days.

**Sampling-Based Method**

Suppose for a network with \( m \) links, the network capacity can be formulated as a variable vector \( \xi = (\xi_1, \xi_2, \ldots, \xi_m) \). Each item \( \xi_i \) represents a certain link capacity with a probability distribution \( p_i \). We can generate a network capacity variable vector with certain realizations, and then propagate the samples through the analysis to produce the mapping \([\xi, F(x, \xi)]\). However, the mapping procedure is usually the most computationally demanding part of a sampling-based uncertainty and sensitivity analysis. For example, supposing a network with 200 links and 3 link statuses, the total number of scenarios is \( 3^{200} \). It is not possible to repeatedly solve the formulation (1) for all capacity realizations. However we can use variance reduction sampling techniques (21) to obtain approximate solutions by selecting subsets of the set \((\xi_1, \xi_2, \ldots, \xi_k)\). This approximate objective, known as a Sample-Average Approximation (SAA) of \( f(x) \) , is then minimized by using a deterministic optimization algorithm. The detailed steps of SAA technique employed in this study can be seen in FIGURE 1. The detailed behavior of this technique and its application can be seen in (21) and (22).

It should be noted that because static assignment is used for each realization of capacity reduction, it assumes that users have perfect information about the current network condition.
Moreover, although in reality it may depend on network topology (e.g. series or parallel links), the capacity of any pair of links are assumed independently distributed in this study. Our consideration is that the proposed framework is mainly used for regional planning. The mathematical formulation, data collection process and analysis are all on a macro-scoopic level. In addition, the incorporation of link correlation in the proposed framework may bring more questions, such as how many upstream links need to be considered, and how to set all the values of degradable factor for all those links. Currently, macro-scoopic data are not sufficient to answer such questions.

FIGURE 1 Representation of SAA Solution Steps

Sensitivity Analysis

Rank Transformation Technique

A number of statistical indices which can be used with SAA procedure are briefly summarized in (21). For a linear relationship, the common statistical indices are correlation coefficients (CC), standardized regression coefficients (SRC), and partial correlation coefficients (PCC). All three indices can provide a criticality strength of linear relationship between $\xi$ and $F(x,\xi)$. For capturing a nonlinear relationship, rank transformation (23) is a well-performed technique proved by (21). By using rank transformation, input data $\xi$ and output result $F(x,\xi)$ are replaced with their corresponding ranks, and then the linear relationship measures can be used. An example rank transformation scheme can be seen in FIGURE 2. A detailed discussion about rank transformation in the context of sensitivity analysis can be found in (23).
Criticality Measures

In this study, sensitivity results obtained with the following criticality measures from (21) are illustrated and compared: rank correlation coefficients (RCCs), standardized rank regression coefficients (SRRCs), and partial rank correlation coefficients (PRCCs).

RCCs

RCCs provides a measure of the strength of the linear relationship between ranked $\xi$ and $F(x, \xi)$. The measure can be mathematically expressed as follows:

$$c(\xi_j, F) = \frac{\sum_{i=1}^{m} (\xi_i - \bar{\xi}_j)(F_i - \bar{F})}{\left[ \sum_{i=1}^{m} (\xi_i - \bar{\xi}_j)^2 \right]^{1/2} \left[ \sum_{i=1}^{m} (F_i - \bar{F})^2 \right]^{1/2}}$$  \hspace{1cm} (4)

Where $\bar{\xi}_j = \sum_{i=1}^{m} \xi_i / m$ and $\bar{F} = \sum_{i=1}^{m} F_i / m$, $m$ is the sampling size.

SRRCs

Regression analysis in this study is formulated as linear models of the following form:

$$\hat{F} = b_0 + \sum_{j=1}^{m} b_j \xi_j$$  \hspace{1cm} (5)

The regression coefficients in equation (5) are determined such that the sums of equation (6) and (7) are minimized, respectively.

$$\sum_{i=1}^{m} (F_i - \hat{F}_i)^2 = \sum_{i=1}^{m} [F_i - (b_0 + b_j \xi_i)]^2$$ \hspace{1cm} (6)

$$\sum_{i=1}^{m} (F_i - \hat{F}_i)^2 = \sum_{i=1}^{m} [F_i - (b_0 + \sum_{j=1}^{m} b_j \xi_j)]^2$$ \hspace{1cm} (7)
The PRCCs equation is related to RCCs equation. Firstly we formulate two regression models:
\[
\hat{\xi}_j = c_0 + \sum_{p=1, p \neq j}^{m} c_p \xi_p \quad \text{and} \quad \hat{F}_j = b_0 + \sum_{p=1, p \neq j}^{m} b_p \xi_p \tag{8}
\]

Then use the new variables \( \xi_j - \hat{\xi}_j \) and \( F - \hat{F} \) to calculate RCC by equation (4). In other words, PRCCs between \( \xi_j \) and \( F \) is the RCCs between \( \xi_j - \hat{\xi}_j \) and \( F - \hat{F} \).

### CASE STUDY

#### Network Description

A case study is performed on the network of Newark, New Jersey. The regional network includes densely-populated, congested locations. We extract the related network from the North Jersey Regional Transportation Model-Enhanced (NJRTM-E) (24) with the appropriate roadways and O-D demands attached to it. When subtracting the transportation network, because of the data collection requirements of accident frequencies, it’s not practical to obtain such data for short roadway segments that are frequently encountered in the NJTRM-E model network. Moreover, in terms of mathematical formulation and sample-based solution method, the link statuses are assumed to be independent. However, in reality the capacity of adjacent links may correlate with each other. Thus, when subtracting the case study network, we attempt to combine several short-distance links in order to incorporate the link capacity correlation observed in reality, and also make the network practical for realistic accident data collection and processing. The resulting network has 28 nodes, 86 links, and 9 O-D pairs. The detailed network can be seen in FIGURE 3.
Analysis Approach
In this case study, the impact of weather and accidents on roadway capacity are considered. For the sampling technique used in SAA, one of the most frequently used sampling methods, Latin Hypercube Sampling (LHS), is used. A detailed convergence and approximation accuracy properties of LHS can be seen in (23). Total users’ travel time under user equilibrium is the performance measure. The general steps of the proposed methodology are as follows:

Step 1: Collection of weather information and accident frequencies
Weather information is obtained from weather stations through national climatic data center (www.ncdc.noaa.gov). The weather information includes precipitation under normal and extreme conditions such as rain or snow. The roadway accident frequency is obtained from the local transportation agencies (25). The detailed accident records include occurrence time, roadway direction, and position by milepost.

Step 2: Generation of actual link capacity distribution
Incorporated with weather and accident database, the actual link capacity distributions are calculated by (2) and (3) using roadway accident data (25). The roadway capacity reduction under different weather and accident conditions are assumed based on literature (27~29).

Step 3: Critical link detection and ranking
By LHS, network capacity realizations are sampled and the critical links are detected and ranked.

Capacity Reduction
Accidents
HCM 2000 (20) summarized related studies and identified some guidelines for capacity reduction due to accidents or incidents such as vehicle break downs. The proportion of capacity available under accident or incident conditions (number of lanes blocked) were summarized and analyzed, which can be seen in TABLE 1.

<table>
<thead>
<tr>
<th>TABLE 1 Freeway Segment Capacity Available Under Incident Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Freeway Lanes by Direction</td>
</tr>
<tr>
<td>-------------------------------------</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
</tbody>
</table>

N/A- not applicable
Source: HCM 2000
**Extreme Weather**

Extreme weather refers to rain, snow, fog and other adverse weather conditions. Recent research (27~29) emphasized the importance of extreme weather intensity in terms of capacity reduction. The weather reduction factors for case study can be seen in **TABLE 2**.

**TABLE 2 Weather Reduction Factors for Case Study**

<table>
<thead>
<tr>
<th>Weather Condition</th>
<th>Rain</th>
<th>Snow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace (&lt;0.01*)</td>
<td>Light [0.01-0.25]</td>
</tr>
<tr>
<td></td>
<td>Trace (&lt;0.01)</td>
<td>Light [0.01,0.25]</td>
</tr>
<tr>
<td>Capacity Reduction</td>
<td>1-5%</td>
<td>5%-10%</td>
</tr>
</tbody>
</table>

* Represented by hourly precipitation

**Results and Discussion**

The detailed critical link detection and ranking can be seen in **TABLE 3**. The *p-value* is used to test the null hypothesis statuses that no relationship exists between the involved variables, versus the alternative of the probability that the strong linear relationship exists. The lower *p-value* means the test results show that the results are less likely to agree with the null hypothesis. In this study, the facility with the lower *p-value* is the more significant or critical link in the degradable transportation network. In **TABLE 3**, with different sensitivity measures of criticality, the results by RCCs, SRCCs and PRCCs are found to be consistent.

**TABLE 3 Critical Link Detection and Ranking in Case Study network**

<table>
<thead>
<tr>
<th>Link Name</th>
<th>RCC</th>
<th>SRCC</th>
<th>PRCC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>p-value</em></td>
<td>Rank</td>
<td><em>p-value</em></td>
</tr>
<tr>
<td>12-19 US-1&amp;9</td>
<td>0.0000</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>21-24 Rahway Ave(I-27)</td>
<td>0.0041</td>
<td>2</td>
<td>0.0000</td>
</tr>
<tr>
<td>1-2 I-78</td>
<td>0.0058</td>
<td>3</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

We compare our criticality measure with the traditional level of service measure in transportation engineering namely, the V/C ratio. **FIGURE 4** shows the congested links in the original case study network (without capacity disruption) in the NJRTM-E model. We can observe that the results are consistent with the critical link detection result. Most of the links with high V/C ratio are critical links, e.g. I-78 and US-1&9. This is acceptable because a high level of service in normal conditions may imply a potentially significant loss for an emergency situation. However, it also shows that the traditional measure of V/C ratio may not capture the effect of route choice decisions when considering links with a degradable status. It is observed that not all links with high level of service in **FIGURE 4** are consistent with the result presented in **TABLE 3**, such as Rahway Ave (NJ-27). The most likely explanation may be the combination of various causes of possible link degradations. Travelers may switch to the local links as the shortest path under the user equilibrium assumption. Especially for congested traffic conditions, when a number of travelers expediently drive out of the dense urban area, even a partial capacity reduction of a local link (due to an incident or extreme weather) may potentially degrade the roadway network performance.
CONCLUSION AND FUTURE RESEARCH

In this study, an analytical framework and efficient solution procedure are proposed for the detection and ranking of critical links considering road capacity uncertainty as a result of day-to-day roadway traffic conditions under uncertainty events impact. Sample-Average Approximation (SAA) methodology is used for addressing network capacity uncertainty and solving the formulated problem. The difference between the proposed methodology and earlier related work is that rather than using a scenario analysis or simply binary status (fail or not) assumption, the proposed framework can be used for capturing realistic day-to-day roadway conditions via multi-status link-capacity analysis. This methodology is potentially useful for improving and extending transportation plans by capturing more realistic conditions, and to offer proactive plans for real-world implementations.

To analyze the proposed methodology, computational analysis is performed using a case study of a portion of the New Jersey network. The value of these realistic results is measured with different statistical methods. The key results demonstrate the potential dangers of ignoring capacity uncertainty in planning and the weakness of traditional V/C ratio for criticality evaluation. However, it should be noted that the proposed methodology and solution procedure is based on static traffic assignment, which assumes that drivers have perfect information about the network condition. It can be more beneficial to investigate the imperfect information impact by using stochastic user equilibrium. Moreover, the proposed methodology may also extend to traffic operation analysis or parameter calibration with micro-simulation tools, e.g. robust intersection signal setting, headway or reaction time calibration for day-to-day roadway conditions.

FIGURE 4 Congested Links (V/C ratio) of Case Study Network in NJRTM-E
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