Impact of Probabilistic Road Capacity Constraints on the Spatial Distribution of Hurricane Evacuation Shelter Capacities

**Mustafa Anil Yazici, M.Sc.** (Corresponding Author)
Graduate Research Assistant,
Department of Civil and Environmental Engineering,
Rutgers, The State University of New Jersey,
623 Bowser Road Piscataway, NJ 08854 USA,
Tel: (732) 445-0579 x119
Fax: (732) 445-0577
e-mail: yazici@eden.rutgers.edu

**Kaan Ozbay, Ph.D**
Associate Professor
Department of Civil and Environmental Engineering,
Rutgers, The State University of New Jersey,
623 Bowser Road Piscataway, NJ 08854 USA,
Tel: (732) 445-2792
Fax: (732) 445-0577
e-mail: kaan@rci.rutgers.edu

Word count: 5744 + 4 Figures + 3 Tables = 7494
Abstract: 241

Submission Date: August 6, 2006

ABSTRACT

The study is focused on determining the change in capacity requirements and desirable shelters locations as a result of link capacity changes during evacuation. A Cell Transmission (CTM) based system optimal dynamic traffic assignment (SODTA) formulation first proposed by Ziliaskopoulos (3) is extended by introducing probabilistic capacity constraints. pLEP method first proposed by Prekopa (8) is used to deal with probabilistic capacity constraints of the proposed stochastic SODTA model. The model captures the probabilistic nature of link capacities due to the impacts of events such as hurricanes and earthquakes that can completely or partially damage highway links. Firstly, a simple single destination example network is studied to show the effectiveness of the proposed model. Then the impact of using stochastic and deterministic link capacities is also analyzed using a simplified multiple-origin multiple-destination version of the Cape May network. The desirable shelter locations are evaluated by letting stochastic SODTA model to assign flows that generates the minimum system-wide travel time. The results show that introducing probabilistic link capacities can adjust the overall flow in the network as well as the shelter utilization. Thus, if the planners consider the predictions of the deterministic model, they may face the risk of not having sufficient food, medicine and other emergency supply in shelters. This paper suggests a more realistic approach to evacuation planning to avoid the inefficient emergency planning that created post-disaster problems in recent major disasters such as Katrina and the Tsunami in South East Asia.
INTRODUCTION
Shelter, as defined in Oxford English dictionary, is “1-a place giving protection from bad weather or danger. 2-a place providing food and accommodation for the homeless. 3-a shielded condition; protection.” Periods with necessary use of shelters are as broad as the definition: evacuations in general but especially hurricane evacuations, earthquake response, war conditions, refugee allocation etc. This paper studies the location and effectiveness of hurricane evacuation shelters with a stochastic programming model. The analysis is focused on determining the desirable location and capacity of shelters, which are less frequently studied compared to their structural and operational aspects. A cell transmission model (CTM) based system optimal dynamic traffic assignment (SODTA) formulation first proposed by Ziliaskopoulos(3) is employed. An example network is studied with possible shelter locations. This model is extended by introducing stochastic cell capacity constraints to capture the probabilistic nature of link capacities due to the impacts of events such as hurricanes and earthquakes that can completely or partially damage highway links. The shelter locations are evaluated by letting stochastic SODTA model to assign flows that generates the minimum system-wide travel time. The number and location of the candidate shelters are assumed to be known a-priori. Then, each possible shelter location is modeled as a destination node that connects to a super destination in order to be consistent with the original formulation suggested by Ziliaskopoulos(3). The destinations are evaluated based on the flows they attract as a result of SO assignment process. The impact of stochastic and deterministic link capacities in terms of the effectiveness of shelters is also analyzed using a simplified version of the complex Cape May network that was recently studied for various real-world Hurricane evacuation scenarios.

LITERATURE REVIEW
Studies and guidelines about shelters are mostly focused on structural and operational issues rather than allocation and they define shelter performance measures under hurricane induced loads(15,16). Operational studies investigate the on-site problems such as first aid, food and water supply etc. There are few other studies that address the effect of shelter locations on the evacuation performance. Sherali et.al.(4) are among the first researchers to address the effect of location. They propose a location-allocation algorithm to determine the best shelter locations among possible combinations, which minimize the congestion related total evacuation time. The proposed model is tested on Virginia Beach network. In Kongsomsaksakul et.al.(2), same problem is solved by employing Stackelberg game to model the interaction between the planning authority and the evacuees. The problem is modeled using a bi-level programming formulation. The planning authority determines the number and locations of shelters that minimize the total network evacuation time, whereas the evacuees simultaneously decide the shelter to go and the route to take within the capacity constraints and location of the shelter. Generic algorithm is used to solve the problem and the findings are applied on Logan network in Utah. Two cases, one being infinite (no capacity constraint) and one being limited shelter capacity is studied. For no capacity constraint case, eight of total 10 shelters are chosen for optimal performance. For limited shelter capacity case, all the candidate shelters are chosen by the evacuees and total travel time naturally increases. For the third case, vehicle occupancy is studied. It is shown that higher occupancy rate.
leads to less total evacuation time, which is also intuitive. Thus, they propose promoting high vehicle occupancy rate during evacuation. Marguis et.al.(1) approach the shelter occupancy issue as a more social concept. They focus on the bus-dispatching scheme to determine the optimal bus allocation for specific pick-up-point-to-shelter routes while maximizing the number of people evacuated over a given time period. This kind of scheme is expected to result in more efficient evacuation, as well as being able to reach as many evacuees as possible that do not have other means of transportation. The evacuation of people with no means of transportation is proven by Hurricane Katrina to be a major issue. The study assumes fixed shelter locations and formulates the problem, but it points out another dimension, which can also be used for determining shelter locations. Although proved otherwise in some hurricane aftermaths, current study assumes that all the evacuees evacuate by individual vehicles and no modeling effort is put to include effect of mass transportation in shelter allocation.

THE MODEL

The evacuees are loaded onto the network from different origins to reach the candidate shelter nodes. System optimal assignment is used to determine the evacuee flows to the shelters that minimize the total system travel time. No capacity is assigned to the shelters and the capacity need of each shelter to maintain the minimum total evacuation time is determined at the end of evacuation period. The change in evacuation performance measures such as average travel time (ATT), clearance times (CT) are also found in case of closing individual shelters. ATT can be considered to be the period during which the evacuee will be exposed to risk until s/he reaches the destination (e.g. shelter). Since CT strongly depends on the loading time of last evacuee onto the network, ATT can give a better idea about the risk exposure. Then the vitality/importance of the shelter is determined by investigating the impact of its absence.

In Sherali et.al.(4) a single period model where volume originating from each zone is assumed to be dissipated at a constant rate throughout the evacuation horizon is proposed. They give a real-life example with constant loading scheme, but as a future extension, they propose a multi period formulation that can use different loading models, such as S-curves. Likewise, Kongsomsaksakul et.al.(2) assign productions that are constant for each time step to each evacuation zone however they do not consider other loading models. Nevertheless, they address another important point, which is the flood risk during evacuation process. Flooding, which is likely to occur as a result of hurricane surges during evacuation, may reduce the link capacities, thus overall evacuation performance. Their study does not include any analysis for flooding conditions, but they suggest using CTM based dynamic traffic assignment to capture the flood dynamics. As studied in Ozbay et.al.(6), the choice of demand model clearly affects the evacuation performance measures. In another study by Ozbay and Yazici(7), evacuation demand model based on the popular S-curves, is studied in detail. It is shown that even when using the same demand generation scheme, changes in model parameters have considerable affect on the evacuation performance. In both studies(6,7), capacity reduction due to external factors, such as flooding, is also analyzed, and the effect of capacity reduction is shown to be an important factor for evacuation studies.

To summarize, in our study a more realistic demand model in conjunction with a model that can capture the flood risk (such as the findings of SLOSH(14)) is used. For
Yazici M.A., Ozbay K.

the traffic assignment, CTM based SODTA formulation proposed by Ziliaskopoulos (3) is employed. The system optimal nature of the assignment is assumed to represent the evacuation conditions accurately since there will be an official evacuation plan implemented by police and other authorities to ensure the most efficient evacuation times. Although it may be claimed that full user compliance can never be achieved, the SODTA provides the “best case” scenario for planning purposes. To capture the probabilistic nature of the flooding, SODTA formulation given in (3) is extended by using probabilistic capacity constraints.

**CTM Based SODTA Model with Probabilistic Capacity Constraints**

Original single-destination LP formulation of Ziliaskopoulos(3) based on CTM assumes deterministic capacity constraints. The complete LP problem construction with detailed explanations can be found in (3). Briefly, the objective of the SODTA problem is to minimize the total travel time in the network, i.e., the travel time experienced by all users of the network. At any time interval \( t \), the travel time experienced by the users of cell \( i \) equals to \( \tau x'_i \), \( x'_i \) being the number of vehicles in cell \( i \) time \( t \). According to the CTM, these users have to stay in this cell for the duration of the time interval. The travel time experienced by all users of the network during time interval \( t \) is

\[
\sum_{\forall i \in \Omega / \zeta} \tau x'_i, \quad \text{because no users are stored at the cell connectors.} \quad \zeta / \zeta, \text{is the set of all cells except the sink cells, because the sink cells do not contribute to the total system travel time. Thus, the total system travel time during the whole assignment period T is}
\]

\[
\sum_{\forall t \in T} \sum_{\forall i \in \zeta / \zeta} \tau x'_i \quad (1)
\]

The SODTA objective is to minimize the function (1), or,

\[
\sum_{\forall t \in T} \sum_{\forall i \in \zeta / \zeta} x'_i \quad (2)
\]

because \( \tau \) was assumed to be one time unit since \( \tau \) can take any positive value without affecting the solution of the LP. The LP problem with complete set of constraints which is a direct extension of the formulation given in (3) are given below (Equations 3-22)

\[
\text{Minimize } \sum_{\forall i \in \zeta / \zeta} \sum_{\forall t \in T} x'_i \quad (3)
\]

Subject to:

Conservation for all cells except source (R) and sink cells (S):
Yazici M.A., Ozbay K.

\[ x_i^t - x_i^{t-1} - \sum_{k \in \Gamma^{-1}(i)} y_{ki}^{t-1} + \sum_{j \in \Gamma(i)} y_{ij}^{t-1} = 0, \forall i \in \zeta \setminus \{ \zeta_R, \zeta_S \}, \forall t \in T \] (4)

Flow inequality constraints for source (R) and ordinary cells (O):

\[ y_{ij}^t - x_i^t \leq 0, \quad \forall (i, j) \in \xi_O \cup \xi_R, \quad \forall t \in T \] (5)

\[ y_{ij}^t \leq Q_j^t, \quad \forall (i, j) \in \xi_O \cup \xi_R, \quad \forall t \in T \] (6)

\[ y_{ij}^t \leq Q_i^t, \quad \forall (i, j) \in \xi_O \cup \xi_R, \quad \forall t \in T \] (7)

\[ y_{ij}^t + \delta_j^t x_j^t \leq \delta_j^t N_j^t, \quad \forall (i, j) \in \xi_O \cup \xi_R, \quad \forall t \in T \] (8)

Flow inequality constraints for sink (S) cells:

\[ y_{ij}^t - x_i^t \leq 0, \quad \forall (i, j) \in \xi_S, \quad \forall t \in T \] (9)

\[ y_{ij}^t \leq Q_i^t, \quad \forall (i, j) \in \xi_S, \quad \forall t \in T \] (10)

Flow inequality constraints for diverging (D):

\[ y_{ij}^t \leq Q_j^t, \quad \forall (i, j) \in \xi_D, \quad \forall t \in T \] (11)

\[ y_{ij}^t + \delta_j^t x_j^t \leq \delta_j^t N_j^t, \quad \forall (i, j) \in \xi_D, \quad \forall t \in T \] (12)

\[ \sum_{\forall j \in \Gamma(i)} y_{ij}^t - x_i^t \leq 0, \quad \forall i \in \zeta_D, \quad \forall t \in T \] (13)

\[ y_{ij}^t \leq Q_i^t, \forall i \in \zeta_D, \forall t \in T \] (14)

Flow inequality constraints for merging (M) cells:

\[ y_{ij}^t - x_i^t \leq 0, \quad \forall i \in \zeta_D, \forall t \in T \] (15)

\[ y_{ij}^t \leq Q_i^t, \quad \forall i \in \zeta_D, \forall t \in T \] (16)

\[ \sum_{\forall i \in \Gamma^{-1}(j)} y_{ij}^t \leq Q_j^t, \forall j \in \zeta_M, \forall t \in T \] (17)

\[ \sum_{\forall i \in \Gamma^{-1}(j)} y_{ij}^t + \delta_j^t x_j^t \leq \delta_j^t N_j^t, \forall j \in \zeta_M, \forall t \in T \] (18)

Mass balance for source cells (R):

\[ x_i^t - x_i^{t-1} + y_{ij}^{t-1} = d_i^{t-1}, \quad j \in \Gamma(i), \forall i \in \zeta_R, \forall t \in T, x_i^0 = 0, \forall i \in \zeta \] (19)

Initial conditions and non-negativity constraints:

\[ y_{ij}^0 = 0, \quad \forall (i, j) \in \zeta \] (20)
Yazici M.A., Ozbay K.

\[ x_i^t \geq 0, \quad \forall i \in \zeta, \quad \forall t \in T \quad (21) \]

\[ y_{ij}^t \geq 0, \quad \forall (i,j) \in \xi, \forall t \in T \quad (22) \]

where;
q: link flow

\[ q_{\text{max}} \]: maximum flow

k: density

\[ k_j \]: jam density

v: link free flow speed

w: backward prorogation speed

\[ \zeta \]: set of cells; ordinary (O), diverging (D), merging (M), source (R) and sink (S).

T: set of discrete time intervals

\[ x_i^t \]: number of vehicles in cell i at time interval t

\[ N_i^t \]: maximum number of vehicles in cell i at time interval t

\[ y_{ij}^t \]: number of vehicles moving from cell i to cell j at time interval t

\[ \xi \]: set of cell connectors; ordinary (O), diverging (D), merging (M), source (R), sink (S).

\[ Q_i^t \]: maximum number of vehicles that can flow into or out of cell i during time interval t

\[ \delta_i^t \]: ratio v/w for each cell and time interval (Assumed \( \delta_i^t=1 \) throughout the analysis)

\[ \Gamma_i(t) \]: set of successor cells to i

\[ \Gamma_i^{-1}(t) \]: set of predecessor cells to cell i

\[ \tau \]: discretization time interval

\[ d_i^t \]: demand (inflow) at cell i in time interval t

This SODTA formulation presented in (3) can be simply given in the following compact standard form shown in equation set (24).
Yazici M.A., Ozbay K.

\[
\min \sum_i \sum_i x_i^i \\
\text{s.t. } A_{eq} \nu = b_{eq}, \quad A \nu \leq b \\
\quad T \nu \leq Q, \\
\quad x_i^i \geq 0, y_{ij}^i \geq 0, \forall (i, j) \in \xi, \forall t \in T
\]  

(23)

where \( \nu = \left[ \begin{array}{c} x_i^i \\ y_{ij}^i \end{array} \right] \) is the vector of system states, and \( Q \) is the deterministic capacity. \( A \) and \( A_{eq} \) stands for the equality and inequality constraints of the original formulation, with corresponding right hand sides \( b \) and \( b_{eq} \). This standard formulation is basically obtained by assigning \( T \nu \) to represent the capacity constraints equations, which are equations (7)-(9), (11)-(13), (15) and (17)-(19). If cell flow and physical capacity are assumed to be probabilistic, the set of relevant constraints can be rewritten as probabilistic constraints for the capacity, as shown in (24).

\[
\min \sum_i \sum_i x_i^i \\
\text{s.t. } A_{eq} \nu = b_{eq}, \quad A \nu \leq b \\
\quad T_i \nu \leq Q \\
\quad P(T_i \nu \leq \phi) \geq p, \\
\quad x_i^i \geq 0, y_{ij}^i \geq 0, \forall (i, j) \in \xi, \forall t \in T
\]  

(24)

The problem formulation shown in equation-25 differs from the standard LP formulation (equation-1) with the flow constraint \( P(T_i \nu \leq \phi) \geq p \), where \( \phi \) is the random variable that represents the probabilistic capacities of the selected cells. Note that \( T \nu \) in original formulation is separated as \( T_i \nu \) and \( T \nu \) to show that both deterministic and probabilistic capacity constraints can be treated simultaneously. A discrete probability distribution is assigned for the capacity and for the solution of this stochastic LP.

**Proposed solution approach for stochastic SODTA problem**

P-Level Efficient Points (pLEP) method proposed by Prekopa (8) can be employed to solve this problem. Below, we give a brief description of this solution technique.

**P-Level Efficient Points**

*Definition:* A point \( z \in Z_p \) is called a p-level efficient point (pLEP) of a probability distribution function \( F \), if \( F(z) \leq p \) and there is no \( y, z, y<z \) such that \( F(y) \geq p \), where \( p \in [0, 1] \).

pLEPs provide discretized set of points, which give the lower bound of a specific probability distribution. For a scalar random variable \( \xi \) and for every \( p \in (0, 1) \), there is exactly one \( p \)-efficient point, namely, the smallest \( z \) such that \( F(z) \geq p \). They are used in the deterministic equivalent of the probabilistic constraint and assure that the constraint will satisfy the given reliability level \( p \). Since the definition of pLEPs form a lower bound
to the given probability distribution, the smaller the $p$, stricter is the assignment. $p=1$ gives the deterministic equivalent for the capacity constraint. As $p$ gets lower, the pLEP points form a lower bound resulting in smaller number to replace capacity in the constraint. In other words, small $p$ value forces the assignment to take less risk in terms of not assigning a capacity that will be higher than the predicted. FIGURE 1 shows the pLEP points for a two dimensional case. Prekopa (9) proposes a recursive algorithm to enumerate the $p$-efficient points for a multidimensional discrete probability distribution. The resulting LP is easily solved by conventional methods after substituting pLEP points in the corresponding constraint.

FIGURE 1 An Example of the set $Z_p$ with pLEPs $v^1 \ldots v^4$ (Source: Dentcheva (10))

A straightforward solution approach is to find all $p$-efficient points and to process all LP problems. Let $v^{(i)}$ is the optimal solution to the $i^{th}$ LP problem with constraint $Tv \geq z^{(i)}$. If $c^Tv^{(i)} = \min_{v^j} c^Tv^{(j)}$, then $v^{(i)}$ is the optimal solution. However, for high-dimensional random vectors the number of pLEPs can be very large and enumeration of those points may not be efficient. For this kind of situations, optimality bounds, or using a dual problem solution as a starting point can be used to decrease the number of pLEPS. For our problem, the network is designed to be tractable with straightforward enumeration of pLEPs, however a detailed discussion on dealing avoiding numerous pLEP enumeration can be found in (10). Details of using pLEPs for stochastic LP problems can be found in (9).

The proposed model with probabilistic constraints are used in (13) and with a simple network and loading scheme, it was shown that probabilistic treatment of roadway capacity reduction can affect the traffic flow considerably. In current study, this fact will be used to determine the effect of this approach on the shelter location and capacity determination. Nevertheless a simple example extracted from (13) is presented below to clarify the usage of the assignment.

**An Illustrative Example for Probabilistic DTA Formulation**

Consider the simple network shown in FIGURE 2. Assume that Cells #7 and #8 are subject to flood risk. These cells are assigned flood probabilities based on the predetermined probability that their individual lanes will be operational during the evacuation, e.g. the probability that 1,2… or all lanes are operational. Practically, this
This kind of discrete approach is quite appropriate for this problem since partial use of a lane is an unrealistic assumption. In other words, a lane is either available, or unavailable. This kind of discrete approach also facilitates the use of pLEP method for the probabilistic capacity constraint given in equation set-25. For calculating pLEP points, the joint cumulative probability is needed. The flood probability for each cell is assumed to be independent, which may sound to be an unrealistic assumption first since those cells represent consecutive roadway portions and the independence assumption can be questioned. However, this assumption causes no loss of generality as far as the problem at hand is concerned. In case of a real application, those probabilities will anyway be determined individually using tools like SLOSH (Sea, Lake and Overland Surges from Hurricanes) which is a computerized model run by the National Hurricane Center (NHC) to estimate storm surge heights and winds. In other words, this independence assumption which is made to avoid mathematical complexities involved while finding joint probability distributions is in fact consistent with the state-of-practice. Moreover, the independence assumption will not impact the meaning of our findings since we are mainly concerned with the impact of the stochastic link capacities on the evacuation times. For this particular network, cell#7, which has 2 lanes are assigned probabilities 0.4, 0.5, 0.1 for 0, 1 and 2 lanes being operational. Likewise, cell#8, which has 4 lanes, is assigned 0.1, 0.2, 0.25, 0.4 and 0.05 for 0, 1, 2, 3 and 4 lanes being operational. The pLEP points for joint probability distribution of those cells are calculated for \( p=0.75 \). As mentioned earlier in this work, these pLEP values are substituted for the capacity values in the deterministic equivalent of the capacity constraint and assure that the probabilistic capacity constraint will hold at the given \( p \) value. The resulting pLEP is found to be (1, 3) which means that to assure \( p=0.75 \), the cell#7 is assigned to have 1 operational lane, and cell#8 to have 3 operational lanes. Then, the network is analyzed with both deterministic and probabilistic constraints. It should be noted that the pLEP points simply provide reduced capacities as given above. Hence, one can argue that reduced capacity assignment can be done regardless of using mathematically sophisticated method like pLEP. However, pLEPs enable the planner to determine a reliability measure \( (p) \) for the given reduced capacity. For different \( p \) values, pLEPs may suggest less or more reduced capacities can be used in LP, however the reliability of grasping the real conditions with the anticipated probabilities also changes. In other words, smaller the \( p \) value shows the less reliability perceived for the anticipated probabilities, or more cautious attitude towards the outcomes of the probabilistic event.

The results show that employing probabilistic analysis adjusts all the flows in the network. The flows on each route are calculated by choosing a cell unique to that path, e.g. cell#9 for Route-1, cell#7 for Route-2 and Cell#3 for Route-3 and summing up the
flow through those cells. The percentage of overall flow through Route-1 increases from 44% to 51%. On the other hand flow on Route-2 decreases from 24% to 17%, whereas flow on Route-3 stays the same at 32%. These results show that using probabilistic constraints alters the flows throughout the network, which may threaten people’s lives in case of an evacuation.

**NUMERICAL EXAMPLE – CAPE MAY COUNTY EVACUATION NETWORK**

Category-4 Cape May Hurricane in 1821 is the last major hurricane to make a direct landfall in New Jersey. According to historical records since 1821, New Jersey is quite safe compared to southern states. However recent changes in world climate and temperature rise warning by scientists increase the importance of evacuation plans for coastal areas. Hence, this study analyzes Cape May County, which is one of the most vulnerable counties of New Jersey. **FIGURE 3** shows the official evacuation routes for Cape May and **FIGURE 4** shows the simplified cell representation of the Cape May evacuation network.

**FIGURE 3 Cape May Evacuation Routes (Source (17))**
Yazici M.A., Ozbay K.

FIGURE 4 Simplified Cell Representation of Cape May Evacuation Network

The analyzed network is a multi-origin multi-destination network, each destination being a shelter location. However, the original SO-DTA formulation is based on single destination. Kalafatas and Peeta (11) suggest that in the evacuation problem, where all destinations are equivalent, a single super-destination cell can be added and connected to all destination cells. Destination cells and the connectors to super-destination are assigned infinite capacity so that there will be no congestion at the destination cells. This suggestion is adopted in the current work.

During evacuation, the vehicles cannot maintain everyday free-flow velocity because of very heavy congestion, thus throughout the analysis, the average evacuation speeds of the vehicles are assumed to be 30 mph. The cell length is set to be 5 miles. Following the requirement of the CTM that a vehicle can traverse at most one cell in one time interval, the time interval is set to be 10 minutes and loading is also performed for each 10 minute interval. Following the Highway Capacity Manual the maximum flow rate is set to be 2160 vehicles per hour per lane and about 150 vehicles are assumed to fit 1 mile road segment. The cells on Garden State Parkway (cell#11, 12, 13, 14, 15, 16) have 4 lanes, whereas the other roads have 3 lanes. Overall network features and cell physical properties are shown in TABLE 1.

TABLE 1 Physical Cell Properties of the Example Network

<table>
<thead>
<tr>
<th>Cell#</th>
<th># of Lanes</th>
<th>Max Flow (veh/τ/ln)</th>
<th>Physical Capacity, N_τ (veh/mile)</th>
<th>Cell Length (miles)</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,5,6,7,20,21,22,23,24</td>
<td>3</td>
<td>1080</td>
<td>450</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>11,12,13,14,15,16</td>
<td>4</td>
<td>1440</td>
<td>600</td>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

τ : Time interval = 10 minutes,

For network loading, S-curve is used following state-of-the-practice in evacuation modeling. Mathematical representation of S-curve is as follows:

\[ P(t) = \frac{1}{1 + \exp[-\alpha(t - H)]} \]  \hspace{1cm} (25)
where $P(t)$ is the cumulative percentage of the total trips generated at time $t$. The “$\alpha$” parameter represents the response of the public to the disaster and alters the slope of the cumulative traffic-loading curve. $H$ is the half loading time; the time at which half of the vehicles in the system have been loaded onto the highway network. $H$ defines the midpoint of the loading curve and can be varied by the planner according to disaster characteristics. The loading parameter choice adjusts the overall performance thus the parameters are kept fixed during all analyses. Specifically, loading parameters are set as $\alpha=0.01$ and $H=6$. According to census data, there are 42148 households in Cape May area and assuming 1 departure from each household, approximately half of the evacuations (21000) are generated from 3 concentrated sources representing the resident and tourist population along the shore.

**Case Studies**

There are 2 basic issues that are studied. First issue is the existence/necessity of a shelter at one of the destinations. Second issue is the effect of flood probability, which may cause decrease in roadway capacity, and eventually change the “favorable” shelter locations. In both cases, capacity of the shelters are under consideration since deciding the location of a shelter is not enough without knowing the number of people that may use that shelter. Maintaining a shelter is further complicated because of emergency supply and response personnel requirements.

Let the problem be the elimination of one shelter out of three shown in FIGURE 4 because of supply logistics and possible problems in finding a sufficient number emergency response personnel that can staff all three shelters. However, let’s also assume that there is a flood risk in the area, especially near the shore, which may cause a specific link to lose part or all of its capacity and consequently alter the selection shelter plan.

First, cell capacities are assumed to be deterministic and constant. Then, the network is analyzed for all possible couples of shelters by eliminating the third one at each iteration. Same procedure is employed but this time with capacity loss probabilities for specifically chosen links. The independence of flood probabilities of cells is again assumed, using reasoning stated above when solving the simple network probabilistic assignment problem.

**Case-1**

For this case, Cells #20, #21 and #22 represent the roadway which is near the shore and covered with water creeks that can be visually seen from aerial photos. Probabilities of the number of lanes that are operational are set to be 0.3, 0.45, 0.20, 0.05 for 0, 1, 2 and 3 lanes respectively for cell#20 and #21. This distribution represents a severe flooding where the fully operational and 1 lane loss probabilities only sum up to 0.25. Cell#22 is assigned probabilities of 0.20, 0.30, 0.45 and 0.05 for 0, 1, 2 and 3 operational lanes, which represent less severe flooding conditions compared to other cells with flood risk. These cells were chosen since the road segments that are represented with these cells are close to the shore and lie in a water-rich area as well. All other cells are assigned fixed, deterministic capacities throughout the evacuation.

First, a complete analysis with all shelters is performed to compare the average evacuation travel time and needed shelter capacities under best conditions. Then each shelter shown in FIGURE 4 are eliminated one-by-one, and the increase travel times and
Yazici M.A., Ozbay K.

Shelter capacity requirements are compared with the complete network where all the shelters are operational. The same procedure is applied for probabilistic road capacity formulation and results are given in TABLE 2. Please note that for the probabilistic assignment, $p$ is set to be 0.75. This $p$ value results in pLEPs which correspond to 1 lane flooding (in other words 2 operational lanes) for all cells with flood risk. Also note that the base scenario is chosen to be the deterministic case and all other performance values are compared with the base scenario.

### TABLE 2 The Results for Deterministic Case and Case-1

<table>
<thead>
<tr>
<th>Abandoned Shelter</th>
<th>ATT* (mins)</th>
<th>ATT Change</th>
<th>Needed Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Operational</td>
<td>66</td>
<td>78</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>(CT**=440)</td>
<td>(CT=450)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S#1</td>
<td>127</td>
<td>169</td>
<td>92%</td>
</tr>
<tr>
<td></td>
<td>(CT=550)</td>
<td>(CT=640)</td>
<td></td>
</tr>
<tr>
<td>S#2</td>
<td>126</td>
<td>169</td>
<td>91%</td>
</tr>
<tr>
<td></td>
<td>(CT=550)</td>
<td>(CT=640)</td>
<td></td>
</tr>
<tr>
<td>S#3</td>
<td>128</td>
<td>128</td>
<td>94%</td>
</tr>
<tr>
<td></td>
<td>(CT=550)</td>
<td>(CT=550)</td>
<td></td>
</tr>
</tbody>
</table>

* ATT: Average Travel Time, ** CT: Clearance Time

As seen in TABLE 2, for the deterministic case, all the shelters as appear to be equal in terms of overall evacuation performance and capacity requirements. The increases in clearance times, average travel times and capacities are equal or very close.

However, when the analysis is done with predetermined flood probabilities, the absence of shelter#1 makes a big difference in evacuation performance. Average travel time increases by 156% whereas the increase in deterministic case is 92%. Also compared to absence of other shelters, shelter#1 is distinguished as the most vital shelter. There is also another point that is of importance other than the location or absence of the shelter. Even if one assumes that all shelters will remain open, probabilistic analysis suggests different capacities for shelters. Results show that the number of evacuees in the shelters will differ by 6%, 10%, -20% for shelter#1, #2 and #3 respectively. These changes are equal up to 1386 evacuees, for instance for shelter #3. If the shelter maintenance aspects are considered, e.g. food-water supply, medical facilities, this difference can change evacuation plans.

### Case-2

For Case-1, it should be noted that the cells, which have flood risk, are the ones near the shore and affect only the evacuees that travel from origin#3 to shelter#3. However, the results are still significant in terms of the impact of the probabilistic analysis on the overall picture. For Case-2, same analysis that is presented in TABLE 2 is repeated by assigning a flood probability to an additional cell. One can also think of this proposed probabilistic capacity decrease as a result of the probability of an accident on the road instead of possible flooding. This perspective may lead us to use probabilistic analysis for links that do not have a major flood risk but high incident risks instead. For
this purpose, cell#23 is assigned a capacity decrease probability. Cell#23 connects shelter#1 to all origins, and is an important cell. The probability distribution, which is same as cell#22 (0.20, 0.30, 0.45, and 0.05 for 0, 1, 2 and 3 operational lanes respectively) is assigned to cell#23. The results of Case-1 and Case-2 are given in TABLE 3. Note that all the increase/decrease comparisons are based on the deterministic base scenario in which all cell capacities are deterministic and all shelters are operational. Same $p$ value as in case-2 is used (0.75) and this again corresponds to 1 lane closure for all cells with flood risk.

### TABLE 3 Results for Case-1 and Case-2

<table>
<thead>
<tr>
<th>Abandoned Shelter</th>
<th>ATT* (mins)</th>
<th>ATT Change</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 Flooded Cells</td>
<td>4 Flooded Cells</td>
<td>3 Flooded Cells</td>
</tr>
<tr>
<td>All Operational</td>
<td>78 (CT=450)</td>
<td>96 (CT=490)</td>
<td>24%</td>
</tr>
<tr>
<td>S#1</td>
<td>169 (CT=640)</td>
<td>169 (CT=640)</td>
<td>156%</td>
</tr>
<tr>
<td>S#2</td>
<td>169 (CT=640)</td>
<td>234 (CT=800)</td>
<td>156%</td>
</tr>
<tr>
<td>S#3</td>
<td>128 (CT=550)</td>
<td>170 (CT=640)</td>
<td>94%</td>
</tr>
</tbody>
</table>

* ATT: Average Travel Time, ** CT: Clearance Time

As seen in TABLE 3 addition of another cell with flood probability alters the overall network performance. For example, the required shelter capacities, when all shelters are operational, change with respect to both deterministic case and Case-1. Shelter#2 receives 959 more evacuees than it did in Case-1, and 1703 more compared to the deterministic case. Demand for shelter#3 increases by 620 compared to Case-1, nevertheless it still stays under the deterministic case demand. For Case-2 shelter#1 demand drops below deterministic case by 937 which was above the deterministic capacity prediction by 642 in Case-1. This shows that not only probabilistic approach changes the overall results, but also a complete probability estimates of flooding for all the cells in the network is essential. Since the network capacities are fully utilized during the evacuation, any change in capacity, especially at merging cells can alter the flows considerably.

The addition of a new flood risk cell also changes the importance ranking of the shelters. In the deterministic case, absence of any of the shelters results more or less in the same overall consequence. In Case-1, shelters #1 and #2 are found to affect the performance of evacuation more than shelter#3. In Case-2, shelter#2 is found to be the most vital shelter and the absence of shelters #1 and #3 are found to have almost the same impact on ATT. In terms of capacity requirements in case of an abandoned shelter, the needed capacity for a shelter can be up to 3029 evacuees. This change is equal to relocating of almost 15% of the total evacuees in Cape May County to operational shelters.
DISCUSSION AND CONCLUSION

In this study, the impacts of incorporating flood probability of cells on shelter locations/importance/capacity are studied. pLEP method first proposed by Prekopa (8) is used to deal with probabilistic capacity constraints of the proposed stochastic SODTA model. The application of the method is first illustrated through a simple example. Then, an example network, which is a simplified cell transmission model of the complex NJ Cape May County, is studied. A state-of-practice loading pattern (S-curve) is used. Our findings show that accounting for flood probabilities, even for links that are not used by all evacuees, can change the system-optimal flows and performance measures, as well as the favorable shelter locations and capacity requirements. Nevertheless, two case studies show that a complete flood risk analysis is also necessary because any new flood probability assignment to a link in an already congested network alters the evacuation pattern considerably. Since shelter allocation is not only building the shelter but also maintaining it, these kinds of shelter allocation and capacity determination models are not sufficient on their own. Other emergency management issues such as medical equipment and personnel, food and water supply, energy supply etc. should also be considered. However, these issues can be dealt efficiently only if planners employ realistic models such as the stochastic SODTA model proposed in this paper which not only captures time-dependent traffic flows but also various stochasticities due the events causing the emergency situation. As shown in this paper, when flooding risk of certain links are incorporated into the model, the demand for shelters changed considerably (highest change being at shelter#2) compared with the predictions of the deterministic model. Thus, if the planners consider the predictions of the deterministic model, they face the risk of not having sufficient food, medicine and other emergency supply in shelter#2. This kind of inefficient emergency planning has already created post-disaster problems in case of major disasters such as Katrina and Tsunami in South East Asia. These recent disasters and post–disaster conditions have only increased the need for better and more realistic planning models along the possible improvements suggested in this paper.

REFERENCES


Yazici M.A., Ozbay K.


