Probabilistic Programming Models for Response Vehicle Dispatching

and Resource Allocation in Traffic Incident Management

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ABSTRACT
This paper introduces mathematical programming models with probabilistic constraints in order to address traffic incident response and resource allocation problems. In incident response problems, introducing the concept of quality of service during a potential incident gives the operator the flexibility to determine the optimal dispatching policy for various situations. We demonstrate that, to minimize the overall response cost, dispatching the closest available service vehicles to the given incident is not necessarily the best strategy if we want to maintain high quality of service in addressing potential incidents. Given the distribution of incidents over a network, we also propose a mathematical model for the resource allocation problem, determining the minimum number of service vehicles allocated to each depot in meeting the requirement of the system reliability. For completeness, relevant concepts and algorithms in stochastic programming are presented briefly. Several examples are included to demonstrate the application of these models to real-world problems. This paper concludes with a summary of results and recommendations for future research.

Keywords: Traffic incident response, mathematical models, probabilistic constraints
INTRODUCTION

Traffic incidents account for approximately 60 percent of the vehicle-hours lost due to congestion annually. It is now widely accepted that these congestion and congestion-related problems can be reduced by the proper use of an efficient incident management system. Since the cost of each response unit is considerable, there is a great need for reliable decision-support models to help evaluate and improve the performance of such systems.

Increased attention in literature has been focused on the incident response problem. The need for improved incident response models and the data available for developing such models are both discussed in Ozbay and Kachroo (1999). Zografos et al. (1993) proposes an analytical framework that can minimize the freeway incident delays through the optimum deployment of traffic flow restoration units (TFRU). The proposed model integrates three modules, namely:

- a districting model to obtain optimal locations of vehicles, to minimize the total average incident response workload per vehicle on freeways, subject to a constraint on the maximum number of available vehicles
- a simulation model that simulates traffic restoration operations
- a dynamic mesoscopic traffic simulation model (KRONOS) that estimates traffic incident delay

The model proposed by Zografos et al. (1993) has proven to be an effective tool that can model and evaluate the effects of deployment of TFRU on overall freeway incident delays.

In a recent paper by Ozbay and Bartin (2003), a complete simulation tool is developed using Arena simulation package, and is used to model and examine the effects of various incident management strategies for the incident management operations in the Washington D.C. beltway network.

Besides computer simulation, mathematical modeling is another frequently used approach to incident management problems. Pal and Sinha (1997) construct a MIP model to determine optimal locations for response vehicles that minimizes the annual response vehicle costs, given the frequencies of incidents at potential sites in the network, and subject to a constraint on the maximum number of vehicles. Recognizing the highly stochastic nature of traffic and incident management operations, Pal and Sinha (2002), introduce a simulation model that can be used for designing a new freeway service patrol, as well as improving the operations of existing programs. Opportunity cost-based models proposed by Sherali et al. (1999) demonstrate that dispatching the closest available vehicle to the site of the current accident is not always the optimal incident response strategy when considering service to anticipated future demands. However, to make this model polynomial-time solvable, the number of response vehicles required by each incident needs to be the same, and each depot has to have the same number of available vehicles. In practice, this is not always true. Due to day-to-day uncertainties, it is always possible to have an insufficient number of vehicles at any given time.

In this paper, we attempt to address the incident management problem, specifically, the interaction between the probabilistic occurrence rates of accidents requiring different levels of resources and the availability of adequate number of incident response units, with a different approach. We introduce the concept of quality of service and propose mathematical programming models with probabilistic constraints to model the stochastic incident response problem. In our model, multiple potential incidents with various demands for response vehicles are allowed, and the number of available response vehicles at each depot is assumed to be non-deterministic. We also assume the probability distributions of the future demand for resources are independent of each other.

In the case of resource allocation, some areas in the transportation network may have a higher probability of experiencing serious incidents than others, due to heavy traffic conditions or complicated geographic characteristics. Thus, more resources should be allocated to those depots that are located closer to risky areas. Given the probabilistic distribution of the demand for resources over the network, we propose a stochastic MIP model to determine the optimum level of resources, and the best way to allocate the resources, over the transportation network.
The remainder of this paper is organized as follows. In the next section, we introduce a traffic incident response problem formulation that captures the uncertainty of the demand for resources and the availability of the resource. A mixed-integer programming (MIP) model with probabilistic constraints and a discrete random variable is proposed to address the dispatching problem with consideration of the requirements at the sites of the future incidents. For the stochastic resource allocation problem, we introduce a mixed-integer nonlinear programming (MINLP) model with probabilistic constraints. The remainder of this section is devoted to the background of mathematical programming under probabilistic constraints, including the key concept of \( p \)-level efficient point for a discrete distribution, and the main ideas for solving this category of problems. In the following section, we provide two examples. The simple incident response example displays the difference between our incident response model and the models that do not consider the likelihood of occurrence of future incidents, and the impact of the quality of service concept on the service vehicle dispatching policy is illustrated. The application of the resource allocation model is demonstrated using a case study that deals with the tow truck allocation on the South New Jersey road network. The paper then concludes with a brief summary of results.

**PROBLEM FORMULATIONS**

The objective of the traffic incident response problem considered in this paper is to minimize the overall response cost under the constraints of limited resources, considering the demands of the future incidents, which is given by a scalar random variable. The ideas are presented in the following order: first, we introduce the concept of quality of service to evaluate the probabilistic constraints and, second, we propose an integer-programming model with probabilistic constraints for the case of stochastic demands at the sites of future incidents. Then, we extend this model to take the variability of available resources into account. Finally, the model for the resource allocation problem is discussed.

To begin, we introduce the mathematical notation used in this paper. Let \( G(N, L) \) be the road network, and \( N \) and \( L \) be its node and link sets respectively. We use \( D \) to denote the set of a special type of nodes, the depots, from which service vehicles are dispatched, let \( F \) denote the node set where incidents occurred, and \( Q \) denote the node site where the future incident might occur. In addition, let \( r_i \) be the number of service vehicles available at node \( i \in D \), \( n_f \) be the number of service vehicles required by the incident occurred at node \( f \in F \), and \( n_q \) be the number of service vehicles required by the future incident might occur at node \( q \in Q \). Note that the service vehicle is an abstract concept for any vehicles that help clear the incident. In practice, it could be tow trucks, ambulances, police cars or fire trucks. In this paper, we consider \( n_q \) as scalar random variables governed by given probability distribution. We let \( \lambda_f \) be the shortest travel time for a service vehicle dispatched from depot \( i \in D \) to the site of an incident at node \( f \in F \). Finally, we use \( x_{if} \) to denote the number service vehicles dispatched from depot \( i \) to an incident at node \( f \), and \( y_{iq} \) to denote the number service vehicles dispatched from depot \( i \) to a future incident that might occur at node \( q \). To help formalize our discussion of the quality of incident response, we offer the following definition:

**Definition-1:** Quality of Service \( p \in (0, 1) \) is used to describe the expectation of the efficiency of incident response operations. All the probabilistic constraints in the mathematical programming formulation should be satisfied with the percentage greater or equal to the value \( p \).

It is worth mentioning that in the incident response problem, zero value of the quality of service for the future incident signifies that the future incident is not considered at all, while the maximum possible resource demand by the future incident is taken into account. In the context of the resource allocation problem, the quality of service makes more sense if we consider it as a measure of the system reliability. Higher quality of service guarantees more incidents can be remedied in a timely manner, thus achieving higher reliability of the transportation system. The effects of quality of service on the
determination of optimal incident response policy and resource allocation strategy are discussed in subsequent sections using real-world examples.

**Dispatching Response Vehicle for Uncertain Resource Demands**

This subsection focuses on how to dispatch response vehicles to incidents if future demand for resources is considered to be non-deterministic. Given the quality of service level \( p \), the proposed model is formulated as follows:

\[
(P1) \quad \text{Minimize } \sum_{i \in D} \sum_{f \in F} \lambda_{if} x_{if} + \sum_{i \in D} \sum_{q \in Q} \lambda_{iq} y_{iq}
\]

Subject to

\[
\sum_{f \in F} x_{if} + \sum_{q \in Q} y_{iq} + s_i = r_i, \quad \forall i \in D \tag{2}
\]

\[
\sum_{f \in F} x_{if} = n_f, \quad \forall f \in F \tag{3}
\]

\[
\prod_{q \in Q} P\left( \sum_{i \in D} y_{iq} \geq n_q \right) \geq p \tag{4}
\]

\[x \geq 0, \ y \geq 0, \ s \geq 0, \ x, y \text{ and } s \text{ are integer numbers.} \tag{5}\]

The objective function minimizes the overall response cost, which includes current response and future response costs. The future response cost is the vehicle-time due to future incidents. By including this part in the objective function, the overall response cost is minimized while the given service quality for future incidents is guaranteed. If we do not consider future incidents, i.e., \( p = 0 \), then any \( y_{iq} \) values will satisfy constraint (4), thus minimizing the objective function forces \( y_{iq} = 0 \). Consequently, there is no future cost in this case. Constraint (2) states that the total number of vehicles dispatched from a depot should be less than the number of vehicles available at that depot, where \( s_i \) are slack variables. Constraint (3) states that the requirement by the current incident must be fully satisfied. Constraint (4) requires the joint probability of meeting the response vehicle requirements at each potential incident site \( f \in F \), should be greater or equal to the given quality of service. \( n_q \) is a discrete random variable whose distribution is known by assumption.

**Dispatching Response Vehicle Under Varying Available Resources**

In addition to the fact that resources demanded by the future incidents are not deterministic, the resource available at the depots for the future incident might be uncertain due to break-down or other unexpected factors. In this case, we can formulate the problem nearly the same as \( P1 \), except that we need to add one more probabilistic constraint, namely,

\[
\prod_{i \in D} P\left( \sum_{f \in F} x_{if} + \sum_{q \in Q} y_{iq} \leq r_i \right) \geq p \tag{6}
\]

Where \( r_i \) is a discrete random variable, \( p \) is the given quality of service.

**Resource Allocation Problem**
Unlike previous problems, the application of probabilistic programming to resource allocation problems is not straightforward. The goal of a resource allocation problem in the context of traffic incident management is to determine the optimal location and optimal fleet size of response vehicles at each depot to minimize the sum of operation cost and investment. The constraint of this problem is that the resources allocated over the network should be enough to adequately address the incidents. We can obtain the distribution of resource demands over the network by analyzing and modeling the historical incident data. Given the distribution of resource demands and the reliability requirement of this incident response system, the resource allocation problem can be formulated as follows, which is a mixed-integer nonlinear programming (MINLP) model with probabilistic constraints.

\[
\text{(P2)} \quad \text{Minimize} \quad \sum_{n \in N} \sum_{f \in F} \lambda_{nf} x_{nf} + c_1 \sum_{i \in N} r_i + c_2 \sum_{i \in N} d_i 
\]

\[\text{Subject to} \]
\[\sum_{f \in F} x_{nf} + s_i = r_i, \quad \forall i \in N \]
\[P\left(\sum_{n \in D} x_{nf} \geq n_f\right) \geq p \]
\[(1 - d_i) \times r_i \leq 0 \]
\[x \geq 0, \ r \geq 0, \ s \geq 0, \ x, \ r \text{ are integers, } d \text{ binary} \]

We use the same notation as previous parts. In addition, let \(c_1\) and \(c_2\) be the unit cost for service vehicles and the construction cost for a single depot respectively. \(d_i\) is binary variables defined as follows.
\[d_i = \begin{cases} 
1 & \text{if node } i \text{ is a depot} (r_i > 0), \\
0 & \text{otherwise.} 
\end{cases} \]

Constraint (9) states the reliability of this incident response system; namely, the joint probability of satisfying the resource requested by each incident should be greater or equal to a given value. Constraint (10) and (11) state the logic relationship between \(d_i\) and \(r_i\). When \(r_i\) is greater than zero, which means node \(i\) is a depot, then \(d_i\) should be one to satisfy the constraint. When \(r_i\) is zero, then \(d_i\) can be anything, but it should choose zero as a value because of the objective function. The objective function is the sum of the response cost and the total investment.

Because of the nonlinear constraint in (10), problem (P2) is not easy to solve. A simplified problem is to determine the number of service vehicles assigned to each depot given the locations of depots, \(D\), and the distribution of resource demands. This simplified problem can be formulated as

\[
\text{Minimize} \quad \sum_{n \in D} \sum_{f \in F} \lambda_{nf} x_{nf} + c_1 \sum_{i \in N} r_i 
\]

\[\text{Subject to} \]
\[\sum_{f \in F} x_{nf} + s_i = r_i, \quad \forall i \in N \]
\[P\left(\sum_{n \in D} x_{nf} \geq n_f\right) \geq p \]
\[x \geq 0, \ r \geq 0, \ s \geq 0, \ x, \ r \text{ are integers} \]

where \(r_i\) is the decision variable.

In the following part, we review the methods to solve these mathematical programming models with probabilistic constraints.
Solution Algorithm

Stochastic programming problems, similar to the ones proposed in previous sections, are formulated based on an underlying deterministic problem in which some of the parameters are random variables. Assume that an underlying deterministic problem is as follows:

\[
\min c^T x \\
\text{subject to } \begin{align*}
&Tx \geq \xi \\
&Ax \geq b \\
x &\geq 0
\end{align*}
\]

where \( \xi = (\xi_1, \ldots, \xi_r)^T \) is a random vector. We require that \( Tx \geq \xi \) shall hold at least with some given probability \( p \in (0, 1) \), rather than for all possible realizations of the right hand side. This leads to the following problem formulation:

\[
\min c^T x \\
\text{subject to } P(Tx \geq \xi) \geq p, \\
&Ax \geq b, \\
x &\geq 0,
\]

where the symbol \( P \) denotes probability.

Because the variables in our model are integers, we give more attention to the stochastic programming with discrete distribution constraints. Before we continue, we shall introduce the concept of a \( p \)-efficient point, defined in Prekopa (1990) as follows.

**Definition-2:** A point \( z \) is called a \( p \)-level efficient point (pLEP) of a probability distribution function \( F \), if \( F(z) \geq p \) and there is no \( y \leq z, y \neq z \) such that \( F(y) \geq p \), where \( p \in (0, 1) \).

To enumerate the \( p \)-level efficient points, Prekopa et al (1998) propose an algorithm, based on which we developed a computer program to enumerate all the efficient points for any multidimensional discrete random variable. Let \( Z_p = \{z^{(1)}, z^{(2)}, \ldots, z^{(N)}\} \) be the set of all pLEP’s, then an equivalent form of the problem (18) is the following:

\[
\min c^T x \\
\text{subject to } \begin{align*}
&Tx \geq z^{(i)} \\
&Ax \geq b \\
x &\geq 0, z^{(i)} \in Z_p.
\end{align*}
\]

A straightforward way to solve (18) is to find all \( p \)-efficient points and to process all corresponding problems, e.g., problem (18) can be solved exactly by the solution of the \( N \) linear programming problems, where the \( i \)th one has the constraint \( Tx \geq z^{(i)} \). If \( x^{(i)} \) is the optimal solution of the \( i \)th problem, and \( c^T x^{(i)} = \min_{z \in Z_p} c^T x^{(j)} \), then \( x^{(i)} \) is the optimal solution of the problem (18). For those problems with small number of pLEP’s and small matrix \( T \), this method is enough. If the random variables in the probabilistic constraints are \( r \)-concave (refer to Dentcheva et al 2000 for detail definition and applications), we can use an equivalent formulation with more convenient structures. Many well-known one-dimensional discrete distributions are \( r \)-concave distributions, to name a few, Poisson distribution, geometrical distribution and binomial distribution (Prekopa, 1995). Binary random vectors and scalar integer random variables are also \( r \)-concave.

If the distribution function of the random variable in problem (18) is \( r \)-concave, then an equivalent representation of (19) can be obtained as
\[
\begin{align*}
\min & \quad c^T x \\
\text{subject to} & \quad Tx \geq t \\
& \quad t \geq \sum_{i \in J} \lambda_i z^{(i)} \\
& \quad Ax \geq b \\
& \quad \sum_{i \in J} \lambda_i = 1 \\
& \quad x \geq 0, z^{(i)} \in Z_p \\
& \quad \lambda_i \geq 0, i \in J, t \text{ is Integer}
\end{align*}
\]

Moreover, if the decision variable \( x \) is integer, then we can simplify (20) further into
\[
\begin{align*}
\min & \quad c^T x \\
\text{subject to} & \quad Tx \geq \sum_{i \in J} \lambda_i z^{(i)} \\
& \quad Ax \geq b \\
& \quad \sum_{i \in J} \lambda_i = 1 \\
& \quad x \geq 0, z^{(i)} \in Z_p \\
& \quad \lambda_i \geq 0, i \in J, x \text{ is Integer}
\end{align*}
\]

It is worth noting that if there is no integer constraint for decision variable \( x \), then formulation (21) cannot be used for problem (18). Actually, the objective value to (21) is the lower bound for the original problem by loosing the constraints of (20). Since in the incident response problem discussed in this paper, the number of service vehicles dispatched should always be integer number, solving the simplified formulation (21) leads to the same solutions to the original problem.

As mentioned above, for some cases, i.e. for multidimensional random vectors \( \xi \), the number of \( p \)-level efficient points can be very large and their enumeration may be difficult. It would be desirable, therefore, to avoid the complete enumeration and search for promising \( p \)-level efficient points only. In this case, the solution method works in such a way that not to find all \( p \)-level efficient points but build them up subsequently by the use of cutting plane method. The detailed discussion is given (Prekopa, 1998), but for the scope of our problem, the method discussed above is sufficient.

**NUMERICAL EXAMPLES**

In this section, we present several examples to demonstrate the applications of the stochastic programming models proposed for incident management. The solution of the incident response problem, namely \( P_1 \), is illustrated using a simple numerical example from a paper written by Sherali et al (1999). The second example used to illustrate the solution of the realistic resource allocation problem, formulated as \( P_2 \) in this paper, is applied to the incident management system in South Jersey. In this example, given the requirement of service quality, we determine the minimum resources allocated to the system in order to maximize the investment on return value.

**Incident Response**

In this subsection, we demonstrate the impact of the future incident via a simple example network taken from Sherali et al (1999). As illustrated in Figure 1, this small network consists of four nodes and four
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arcs. Node 1 and 2 represent depots, while node $f$ and $v$ represent the site of incidents. The shortest travel times between depots and sites of incidents are $\lambda_{1f} = 54$, $\lambda_{2f} = 27$, $\lambda_{1v} = 44$ and $\lambda_{2v} = 18$. The number of service vehicles available at each depot is deterministic and fixed, which is assumed to be 5 service vehicles and 3 service vehicles at depot 1 and depot 2 respectively.

![Figure 1 Example network based on Sherali (1999)](image)

Assume the incident occurred at node $f$, requiring 2 service vehicles, and the possible requirement of service vehicles by an additional incident might occur at node $v$, is a random variable which is governed by the scalar distribution shown in Table 1. We need to determine the optimal service vehicle dispatching policy in the term of minimizing the overall response cost.

<table>
<thead>
<tr>
<th>Number of service vehicles</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.6</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

If we neglect the potential demand of service vehicles at node $v$, then dispatching two service vehicles from depot 2 to the site of incident, $f$, would be the optimal response strategy. On the other hand, if we consider the potential demand to “some” extent, using the knowledge introduced in previous sections, the underlying mathematical model for this small example can be formulated as follows:

Min $54x_{1f} + 27x_{2f} + 44y_{1v} + 18y_{2v}$

s.t.

$x_{1f} + y_{1v} \leq 5$

$x_{2f} + y_{2v} \leq 3$

$x_{1f} + x_{2f} = 2$

$P(y_{1v} + y_{2v} \geq n_v) \geq p$

$x_{1f}, x_{2f}, y_{1v}, y_{2v}$ are nonnegative integers

, where $0 \leq p \leq 1$ is the given probability value. $p = 0$ means the potential demand is not considered, and $p = 1$ means the worst case conditions of the potential incident would be covered by the resulted dispatching strategy. Table 2 gives the optimal number of vehicles dispatched for different $p$ levels, which is visualized in Figure 2.
Table 2 Optimal dispatching policies at different $p$ levels

<table>
<thead>
<tr>
<th>$P$</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1f}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$x_{2f}$</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$y_{1v}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$y_{2v}$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Objective function value

| | 54 | 72 | 98 | 124 | 168 |

Figure 2 Number of service vehicles increases with $p$ values

Table 2 and Figure 2 display that the total number of service vehicles involved in the incidents increases while we raise the $p$ values. Note that when the possibility of the potential incident is excluded or only consider the possibility of the low demand of the potential incident, it is straightforward that the optimal solution would be dispatching the closest available service vehicles to the site of the incident. In this example, all of the requested two vehicles would be dispatched from depot 2 to the indent occurred at node $f$. As the $p$ level increases, dispatching the closest available response vehicle could not be a wise choice. For instance, at $p = 0.8$, according to the optimal results shown in Table 2, among the two service vehicles requested by the occurred incident at node $f$, only one of them is dispatched from the closest depot (depot 2), although there are enough service vehicles available at depot 2 in the current setting. The service vehicles that are not dispatched to node $f$ are saved for the potential demand by future incident at node $v$, since dispatching service vehicles from depot 2 to node $v$ is much cheaper than dispatching from depot 1. If we ignore the risk of the potential incident, stick to the origin optimal policy. After sending the two closest vehicles from depot 2 to meet the requirement of the occurred incident, if an incident requested two service vehicles does happen at node $v$ (the probability is 0.1), we have no choice but send the last one available vehicle from depot 2 and another vehicle has to be sent from the distant node 1 to satisfy the demand at node $v$. This results in a total response cost as high as 126, compared to the obtained optimal value, 98. As the $p$ value increases, more and more service vehicles at depot 2 are saved for the future incident.

Resource Allocation Problem

In this example we consider the South New Jersey road network which consists of seven major roads depicted in Figure 3. Tow truck depots are constructed along these routes to clear incidents. The number of tow trucks sent to the site of the incident is determined by the attributes of the incident. If the
number of tow trucks requested by the incident exceeds the number of the available tow trucks in the nearest depot, then the rest of the trucks would be sent from the next nearest depot with idle trucks. To maximize the return on investment, we need to determine the minimum number of tow trucks allocated to each route, while satisfying the given requirement of the quality of service.

![Figure 3 South New Jersey roadway network](image)

The distribution of the demand for the tow trucks is based on the analysis of the traffic incident data in South New Jersey.

**Table 3** Number of incidents on the major routes in South New Jersey (Year 2000 ~ Year 2001)

<table>
<thead>
<tr>
<th>Category (Average number of tow trucks requested)</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAZMAT (4)</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>Vehicle-fire (3)</td>
<td>1</td>
<td>1</td>
<td>11</td>
<td>5</td>
<td>2</td>
<td>27</td>
<td>2</td>
</tr>
<tr>
<td>Weather-related (1)</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Disablement-No Blocked Lanes (1)</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Disablement-Blocked Lanes (2)</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>6</td>
<td>1</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>MVA-Day Time-No Blocked Lanes (1)</td>
<td>21</td>
<td>21</td>
<td>22</td>
<td>36</td>
<td>30</td>
<td>94</td>
<td>8</td>
</tr>
<tr>
<td>MVA-Day Time-Blocked Lanes (2)</td>
<td>12</td>
<td>14</td>
<td>24</td>
<td>25</td>
<td>12</td>
<td>81</td>
<td>10</td>
</tr>
<tr>
<td>MVA-Night Time-No Blocked Lanes (1)</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>MVA-Night Time-Blocked Lanes (2)</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>12</td>
<td>2</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>51</td>
<td>49</td>
<td>76</td>
<td>99</td>
<td>62</td>
<td>264</td>
<td>35</td>
</tr>
</tbody>
</table>
By categorizing the traffic incidents that occurred in years 2000 and 2001 into Table 3, and assuming the average number of tow trucks demanded by each category, as listed in the first column of Table 3, we achieve the scalar probability values which is presented in Table 4. The average traveling time along each route with and without incidents is obtained by a traffic simulation software package developed by ourselves. The results are also shown in Table 4. Here we don’t consider the impact of incidents on the travel time on each link.

Table 4 Average travel time and distribution of requested tow trucks on each route

<table>
<thead>
<tr>
<th>Route #</th>
<th>Route</th>
<th>Average travel time (s) without incidents</th>
<th>Average travel time (s) with incidents</th>
<th>Distribution of requested tow trucks</th>
</tr>
</thead>
<tbody>
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<td></td>
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<tr>
<td>1</td>
<td>US 30</td>
<td>754</td>
<td>6041</td>
<td>.647</td>
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<td>2</td>
<td>NJ 38</td>
<td>286</td>
<td>313</td>
<td>.571</td>
</tr>
<tr>
<td>3</td>
<td>NJ 42</td>
<td>554</td>
<td>616</td>
<td>.342</td>
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<tr>
<td>4</td>
<td>I-76</td>
<td>242</td>
<td>4301</td>
<td>.484</td>
</tr>
<tr>
<td>5</td>
<td>US 130</td>
<td>1246</td>
<td>1666</td>
<td>.677</td>
</tr>
<tr>
<td>6</td>
<td>I-295</td>
<td>773</td>
<td>5138</td>
<td>.421</td>
</tr>
<tr>
<td>7</td>
<td>I-676</td>
<td>305</td>
<td>4426</td>
<td>.429</td>
</tr>
</tbody>
</table>

The road network which is simplified further into a graph, where by using each node, represents one route as shown in Figure 4. A link between two nodes implies there is an intersection that connects these two routes.

Figure 4 Simplified graph representation of the South New Jersey road network

We assume the time for the tow truck traveling from its depot to the site of the incident on the same route is the average travel time of this route under the impact of incidents. If tow trucks that belong to another route are sent from the depot, then the total traveling time of the tow truck would be the summation of the average travel time of each node along the shortest path from the depot to the site of the incident and the average traveling time with incidents of the ending nodes. Based on these assumptions and the shortest path algorithm, the traveling time between every two nodes are obtained as in Table 5.
Table 5  Average travel time between each pair of nodes (seconds)

<table>
<thead>
<tr>
<th></th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
</tr>
</thead>
<tbody>
<tr>
<td>US 30</td>
<td>6041</td>
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<td>1917</td>
<td>5360</td>
<td>2420</td>
<td>5892</td>
<td>5180</td>
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<tr>
<td>NJ 38</td>
<td>6327</td>
<td>313</td>
<td>2203</td>
<td>5646</td>
<td>1952</td>
<td>6178</td>
<td>5466</td>
</tr>
<tr>
<td>NJ 42</td>
<td>7142</td>
<td>2168</td>
<td>616</td>
<td>4855</td>
<td>1765</td>
<td>5692</td>
<td>5222</td>
</tr>
<tr>
<td>I-76</td>
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<td>1614</td>
<td>858</td>
<td>4301</td>
<td>1908</td>
<td>5380</td>
<td>4673</td>
</tr>
<tr>
<td>US 130</td>
<td>7287</td>
<td>1559</td>
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<td>5074</td>
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<td>5441</td>
</tr>
<tr>
<td>I-676</td>
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<td>1163</td>
<td>4606</td>
<td>2213</td>
<td>5685</td>
<td>4426</td>
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</tbody>
</table>

Let $x_{ij}$ be the number of tow trucks sent from the depot on route $i$ to the site of incident occurred on route $j$, and $t_{ij}$ be the according travel time. The objective function of this problem can be presented as

$$\text{MIN} \sum_{i=1}^{7} \sum_{j=1}^{7} x_{ij}t_{ij} + C \sum_{i=1}^{7} \sum_{j=1}^{7} x_{ij},$$

where $C$ is the cost of a single tow truck.

If the service guarantee level $p = 0.9$, referring to the algorithm presented in previous part, the efficient points are obtained in Table 6.

Table 6 Efficient points with $p = 0.9$

<table>
<thead>
<tr>
<th>Point number</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>4</td>
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<td>4</td>
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<td>3</td>
</tr>
</tbody>
</table>

Let $n_{ij}$ be the number of tow trucks on route $j$ according to $i$th efficient points, and let $\lambda_{i}$ be the coefficients of $i$th efficient point which satisfies $\sum_{j=1}^{15} \lambda_{j} = 1$. Then the main constraints of this optimization problem are
\[
\sum_{i=1}^{7} x_{ij} \geq \sum_{j=1}^{15} \lambda_j n_j \\
\sum_{j=1}^{15} \lambda_j = 1
\]

Solving this problem, we achieve the optimal resource allocation strategy. Similarly, we can compute the optimal number of tow trucks assigned to each routes for different \( p \) values which is summarized in Table 7.

<table>
<thead>
<tr>
<th>( p )</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
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<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
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<tr>
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<td>3</td>
<td>4</td>
<td>4</td>
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<td>25</td>
</tr>
</tbody>
</table>

**Figure 5** Optimal number of tow trucks assigned to each routes for different \( p \) levels

Figure 5 visually depicted the results presented in Table 7. Without surprise, to improve the quality of service more tow trucks assigned to the routes is required. However, it is worth to note that the number of tow trucks assigned to each route does not increase simultaneously when we demand higher quality of service. For instance, to raise the \( p \) value from 0.5 to 0.9, US30 keeps the same number of tow trucks while I676 increase its tow trucks rapidly. This happens as a result of the overall travel time from US130 being much longer than I676. To cover the future the incidents, assigning more tow trucks to I676, instead of US30, is a wise choice.

**SUMMARY AND DISCUSSION**

In this paper, we proposed mathematical programming models with probabilistic constraints to stress the future service demands in the incident response problem. The probabilistic constraints are derived from the number of service vehicles requested by the future incident and the number of service vehicles
available at the depots fluctuates. Quality of service is introduced to measure to what extent the potential incident is considered in determining the response policy for the current set of incidents. High quality of service means the future incident plays an important role in the choice of response policy for the current set of incidents, and vice versa. To put it in another way, high quality of service for the future incidents is a conservative practice, while low quality of service means high risk. If the future incident really happens, it will result in much higher response cost compared to a high quality of service value. We assume the distributions of the resource requested by the future incidents are known, and independent from each other, from which efficient points could be obtained for a given requirement of the quality of service. The impact of the quality of service is also studied in this paper. With no doubt, the total response cost raises as the quality of service increase, since more service vehicles are required to guarantee higher quality of service. In this case, although the number of service vehicles at each depot involved in the incident response is non-decreasing, but the destination of the service vehicles dispatched to could change. To lower the overall response cost, current incidents sometimes need to make sacrifices for the future incidents by not using the closest available resources.

The resource allocation problem with probabilistic constraints for the incident management system is also addressed in this paper. Given the distribution of the incidents on a transportation network, we constructed a probabilistic programming model to determine the minimum number of service vehicles allocated to each depot, in order to meet the reliability requirement of this system. This offers the planners the capability of considering inherent stochasticities of the system when they attempt to determine the solution that maximizes the return on the investment, while remaining within certain budget constraints. In the case study of the South New Jersey road network, we derived the distribution of the demand for tow trucks along the eight major roads from the analysis of two years of incident data in this area. Based on the distributions, mathematical models were constructed, and the minimum number of tow trucks allocated to each major road for various reliability requirements was obtained.

The mathematical programming models with probabilistic constraints discussed in this paper are solved using the concepts and algorithms reviewed in previous sections. We developed a computer program to enumerate all of the efficient points for any multidimensional scalar random variables. Thus, our incident response model is not limited to a single future incident. Moreover, the number of service vehicles requested by each incident can be different, and the number of vehicles available at each depot is also allowed to vary.

Acknowledgment

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Reference

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